Ch. 7 Exponents

multiply (same base)	ADD exponents (mult. is more = add)	Ex. $3^2 \cdot 3^3 = 3^5$ $a^{-2} \cdot a^4 = a^2$
divide (same base)	SUBTRACT exponents hint: if you divide candy you get less (-)	$\frac{4^3}{4^1} = 4^2 = 16$ $\frac{x^5}{x^2} = x^{5-2} = x^3$
power to a power	MULTIPLY (extra powerful) most if you x	$(3^2)^4 = 3^8$ $(a^2)^3 = a^6$
zero power	equals 1	$a^0 = 1$ $3^0 = 1$
Negative exponents are improper	Move below or above fraction bar to change sign	$x^{-7} = \frac{1}{x^7}$ can't leave numerator blank, so insert 1

Other Examples:

$\frac{25x^6}{5x^2}$ • divide #s • where do you have larger exponent, put it there $\rightarrow \frac{5x^{6-2}}{1} = 5x^4$

$\frac{3x^{-4}y^3}{12xy^{-5}} = \frac{1y^8}{4x^5}$ ← 3−−5, 1−−4; 2 negs in a row turn to a positive

$[(2^3)^2]^4 = 2^{24}$ mult. all exp.

$(3x^2)(2x^3y) = 6x^5y$ mult. whole #s add exponents if like bases

Note: exponent applies to everything in parenthesis. $\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}$

Note: exponent applies to what it is next to.

Ex. $-2^2 =$ ← exponent next to 2, not the negative. $\pm 2 \cdot 2 =$ $\boxed{-4}$

Ex. $(-2)^2 =$ ← exponent next to parenthesis, mult. all within ( ) $(-2)(-2) =$ $\boxed{4}$

Rational Exponents

$b^{\frac{1}{2}} = \sqrt{b}$ | $16^{\frac{1}{2}} = \sqrt{16} = 4$ | $17^{\frac{1}{2}} = \sqrt{17}$

$(5w)^{\frac{1}{2}} = \sqrt{5w}$ | $x^2 = 25$, so $x = \pm 5$

$\sqrt[3]{27} = \sqrt[3]{3 \cdot 3 \cdot 3} = 3$

$a^{\frac{1}{n}} = \sqrt[n]{a}$ | $8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = (\sqrt[3]{4 \cdot 4 \cdot 4})^2 = 4^2 = 16$

$25^{x-1} = 5$
$(5^2)^{x-1} = 5$
$5^{2x-2} = 5$ (divide by 5)
$2x - 2 = 1$ (add 2)
$2x = 3$ (divide by 2)
$x = 3/2$

Scientific Notation

3.45×10^{-3} ← if negative move decimal to the left. Add zero placeholders. [0.00345]

2.51×10^3 ← if positive move decimal to right. [2510]

2,154 = 2.154×10^3 move decimal so only 1 digit is to left. Add how many places you moved decimal.

0.521 = 5.21×10^{-1} If # is less than 1, then exponent is negative.

Ch. 8/4 Slope

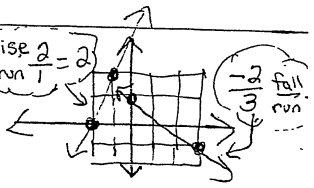
From 2 points: (2, −3) (−1, 4) insert in formula $\frac{y-y}{x-x}$

Ex. $\frac{4 - -3}{-1 - 2} = \frac{4+3}{-3} = \boxed{\frac{7}{-3}}$

Note: two negatives make a positive

From a graph:

- begin at the point to the left
  - if uphill count $\frac{\text{rise}}{\text{run}}$ (positive slope)
  - if downhill count $\frac{\text{fall}}{\text{run}}$ (negative slope)



From a table:

- Find $\frac{\text{change in } y}{\text{change in } x}$ abbreviation $\frac{\Delta y}{\Delta x}$

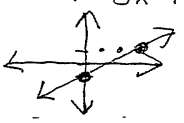
x	2	4	6
y	12	18	24

(+2, +2; +6, +6) $\frac{\Delta y}{\Delta x} = \frac{6}{2} = \boxed{3}$

Graphing

Slope Intercept Form $y = mx + b$ (m = slope, b = y-intercept (where line intersects the y axis))

1st plot y intercept, 2nd from that point apply slope. Ex. $y = \frac{2}{3}x - 1$



Standard Form:

$Ax + B = C$ Ex. $2x + 3y = 12$

x + y are on the left of equals sign

rules: no fractions (mult. by denominator), no leading negative (mult by −1)

1st make x equal to 0, solve for y
2nd make y equal to 0, solve for x
3rd plot these intercepts (0, 4) (6, 0)

x	y
0	4
6	0

$2x + 3y = 12$
$2(0) + 3y = 12$
$\frac{3y}{3} = \frac{12}{3}$
$y = 4$

Write equation in slope intercept form for a line that passes through (2, 1) with a slope of 3

1st – insert points and slope and solve for b

2nd – insert slope and y-intercept into formula

Ex. $y = mx + b$
$1 = 3(2) + b$
$1 = 6 + b$
$-6 \quad -6$
$-5 = b$

The slope was from the problem → $y = mx + b$ → $\boxed{y = 3x - 5}$ ← y-intercept you found above

Write an equation in standard form that passes through points (−1, 7) and (8, −2)

1st – Find slope using $\frac{y-y}{x-x}$ Ex. $\frac{-2-7}{8--1} = \frac{-9}{9} = -1$

2nd Choose either point and find b

3rd Chg to Standard (see above problem)

$y = mx + b$
$-2 = -1(8) + b$
$-2 = -8 + b$
$+8 \quad +8$
$6 = b$

$y = mx + b$
$y = -1x + 6$ (Change to standard form)
$x + y = 6$

Sci. Notation X

• mult #s • add exp

$(3.5 \times 10^{-3})(7 \times 10^5)$
24.5×10^2 ← move
2.45×10^3 ← Chg to proper scientific notation. Think of balance scale.

made this side smaller, moved dec. 1 place, so move other side 1 larger (opposite)

Division

• divide #s • subtract exponents

$\frac{3.066 \times 10^8}{7.3 \times 10^3}$
42×10^5 ← balance scale
4.2×10^4

Exponential Functions

Graph $y = \left(\frac{1}{3}\right)^x$ $y = 3^x$

• Make table. Use any positive + negative #'s

x	$(\frac{1}{3})^x$	y
−2	$(\frac{1}{3})^{-2}$	9
0	$(\frac{1}{3})^0$	1
2	$(\frac{1}{3})^2$	$\frac{1}{9}$

• Graph

Is this exponential growth?

x	0	5	10	15
y	64	32	16	8

✓ for multiplication pattern

$\times \frac{1}{2} \quad \times \frac{1}{2} \times \frac{1}{2}$

Growth

$y = a(1 + r)^t$ ← time in periods; rate as decimal; initial amt; final amt

Ex. College costs 10,850 and increases 5% per yr. Find cost in 3 yrs

$y = 10850(1 + .05)^3$

Solve using PEMDAS

Elimination

- Find variable to eliminate
- solve
- put answer in and solve for other variable

$2x + 3y = 11$
$-2x + 9y = 1$
$12y = 12$
$\frac{12y}{12} = \frac{12}{12}$
$y = 1$

$2x + 3y = 11$
$2x + 3(1) = 11$
$2x + 3 = 11$
$-3 \quad -3$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$
$\boxed{(4, 1)}$

Prepare to eliminate – Find LCM so a term will cancel out. Then distribute and solve for each variable

LCM 2: $3x - 2y = 10$ → $6x - 4y = 20$
LCM −3: $2x + 3y = -2$ → $6x - 9y = 6$
distribute
$-13y = 26$
$y = -2$

Polynomials

Ex.	Degree (# of largest exponent)	Name
$4x$	1	monomial
-5	0	
$3x^2+5b$	2	bi-nomial
$2x^4+x^3-x$	4	tri-nomial

Add/Subtract Polynomials

Ex. $(3x+x^2-4)+(2x^2-2+3x)$

line up like terms and add the coefficients. Variable stays the same.

(x^2+3x-4)
$+(2x^2+3x-2)$

$3x^2+6x-6$

Ex. $(4-2x^2+4x)-(3x^2+5)$

- Change all signs after the subtraction sign to the opposite.

$(-2x^2+4x+4)$
$+(-3x^2\quad -5$ Then add like above.

$-5x^2+4x-1$

Multiply • mult. #s and add exponents

Ex. $-3x^2(4x^2-x+2)=$

$-12x^4+3x^3-6x^2$

Multiply Binomials (distribute)

$(x-2)(3x+4)=$

$x(3x+4)-2(3x+4)$ • distribute

$3x^2+4x-6x-8$ • multiply #s • add exponents • combine like terms

$3x^2-2x-8$

$(2x+3)(4x^2-2x-4)$ • distribute to all 3 terms • mult #s • add exponents

Special Products – Short Cuts – Memorize

Add

$(a+b)^2=(a+b)(a+b)$

$=a^2+2ab+b^2$ (square; 2 times both terms; square)

Ex. $(x+4)^2=$
$(x+4)(x+4)=$
$x^2+8x+16$

Subtract

$(a-b)^2=a^2-2ab+b^2$ like above except middle term is negative

Ex. $(6p-1)^2$ (square; $-2(6\cdot1)$; square)

$36p^2-12p+1$

One positive + one negative

$(a+b)(a-b)=a^2-b^2$

- Square 1st term
- Square 2nd term
- No middle term

Ex. $(2x+3)(2x-3)=$
$4x^2-6x+6x-9$ always cancel out, so

$4x^2-9$

Zero Product Property (set = into two and solve)

$(2x+3)(x-4)=0$

$2x+3=0$ $\quad x-4=0$
$-3\ -3$ $\quad +4\ +4$

$\frac{2x}{2}=\frac{-3}{2}$ $\quad x=4$

$x=-\frac{3}{2}$

x^2+x-42

Factor, then set to zero

$(x-6)(x+7)=0$

$x-6=0$ $\quad x+7=0$
$+6\ +6$ $\quad -7\ -7$

$x=6$ $\quad x=-7$

Quadratics MEMORIZE

$y=ax^2+bx+c$

$y=2x^2+3x+4$ ← y-intercept

Always have a variable squared x^2

if positive (smile) → minimum point

if negative (frown) → maximum point

position of axis of symetry

large # = narrow graph

small # = wide graph

(ignore sign, only use absolute value of the number.)

Ex. Order narrow to wide: $-4x^2$ $\frac{1}{4}x^2$ x^2

order = $\frac{1}{4}x^2$ x^2 $-4x^2$

↖ largest number since you use absolute value

Vertex Formula

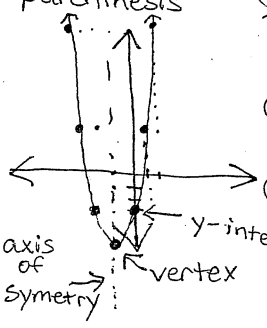
$\frac{-b}{2a}$

Graph

$y=2x^2+4x-3$

terms a b c

Hint: always insert negatives in parenthesis



① insert terms into vertex formula.

$\frac{-b}{2a}$ $\frac{-4}{2(2)}=\frac{-4}{4}=-1$ this is x term. $(-1,\ y)$

② insert x term into problem to solve for y

$y=2(-1)^2+4(-1)-3$

$y=2(1)-4-3$

$y=-5$ Vertex $=(-1,-5)$

③ plot vertex and make dashed line for axis of symetry

④ plot y-intercept and reflection over axis of symetry −3

⑤ Now choose 2 other numbers to put into equation and solve.

x	$2x^2+4x-3$	y
1	$2(1)^2+4(1)-3$	3
2	$2(2)^2+4(2)-3$ $8+8-3$	13

⑥ Graph those points and reflections.

(1, −3) (2, 13)

⑦ Do √'s • is graph going the correct direction $2x^2+4x-3$ ↝ positive = ↜ • is y-intercept in correct place?

domain = {all real #'s} (x)

range = {y ≥ −5} (y)

Ex. Graph $y=\frac{1}{3}x^2$ ← note y-intercept = 0 since there is no C term

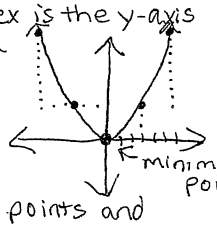
① graph y-intercept

② no b term, so the vertex is the y-axis. No need to use formula

③ Find 2 other points

FRACTIONS – use LCM (least common multiple of denominator)

x	$\frac{1}{3}x^2$	y	
3	$\frac{1}{3}\cdot(3^2)$	3	(3, 3)
	$\frac{1}{3}\cdot9=\frac{9}{3}$		
6	$\frac{1}{3}\cdot\frac{36}{1}=$	12	(6, 12)



④ graph points and their reflection

No b term

$-2x^2+4$

Negative so parabola is down. (Frown)

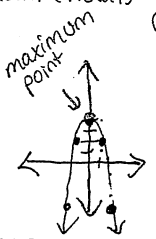
① rewrite in ax^2+bx+c format

$-2x^2+0x+4$

② now use formula to find vertex

$\frac{-b}{2a}$ $\frac{-0}{-2(2)}=0$ $(0, 4)$

then find y by inserting 0 into problem



x	y
1	2
2	−4

$-2x^2+4$

$-2(0)^2+4$

How many solutions (or roots) (count number of times graph crosses x-axis)

2 solutions 1 solution No solution